

Use Pascal's Triangle to expand and simplify $(n^3 - 2r^4)^5$.

SCORE: ____ / 5 PTS

$$(n^3)^5 + 5(n^3)^4(-2r^4) + 10(n^3)^3(-2r^4)^2 + 10(n^3)^2(-2r^4)^3 + 5(n^3)(-2r^4)^4 + (-2r^4)^5$$

$$= \underbrace{n^{15}}_{\textcircled{\frac{1}{2}}} - \underbrace{10n^{12}r^4}_{\textcircled{1}} + \underbrace{40n^9r^8}_{\textcircled{1}} - \underbrace{80n^6r^{12}}_{\textcircled{1}} + \underbrace{80n^3r^{16}}_{\textcircled{1}} - \underbrace{32r^{20}}_{\textcircled{\frac{1}{2}}}$$

Using the formulae for the sums of the powers of integers in your textbook, find the sum $\sum_{n=1}^{240} (2 - 5n + 3n^2)$. **SCORE: _____ / 4 PTS**

Your final answer may **NOT** use ... **NOR** $\sum$. It may use $+$, $-$, $\times$, $\div$. (It does **NOT** need to be simplified into a single number.)

$$\sum_{n=1}^{240} 2 - 5 \sum_{n=1}^{240} n + 3 \sum_{n=1}^{240} n^2$$

$$= \underbrace{240 \cdot 2}_{\textcircled{\frac{1}{2}}} - 5 \cdot \underbrace{\frac{240(241)}{2}}_{\textcircled{1\frac{1}{2}}} + 3 \cdot \underbrace{\frac{240(241)(481)}{6}}_{\textcircled{2}}$$

FJ started a 30 day treatment program which involved daily injections of a medication. The first day's injection was 12 mg, and each subsequent day's injection was 5% less than the previous day's injection. Find the total amount of the injections. **SCORE: _____ / 5 PTS**

Your final answer may **NOT** use ... **NOR** $\sum$. It may use +, -, $\times$, $\div$ and powers. (It does **NOT** need to be simplified into a single number.)

a_n = AMOUNT OF INJECTION ON n^{TH} DAY

$$a_1 = 12$$

$$a_2 = 12(1 - 0.05) = 12(0.95)$$

$$a_3 = 12(0.95)^2$$

$\vdots$

$$a_n = 12(0.95)^{n-1}$$

$$\text{TOTAL} = \frac{\overset{\textcircled{1}}{12} \overset{\textcircled{2\frac{1}{2}}}{(1 - 0.95^{30})}}{\underset{\textcircled{1\frac{1}{2}}}{1 - 0.95}}$$

Find the rational number representation of the repeating decimal $0.2\overline{63}$ using the method discussed in lecture.

SCORE: ____ / 6 PTS

NOTE: Only the 63 is repeated.

$$\underline{0.2 + 0.063 + 0.00063 + 0.0000063 + \dots}$$
$$= \frac{2}{10} + \frac{0.063}{1 - \frac{1}{100}} \quad (1)$$

$$= \frac{2}{10} + \left[\frac{\frac{63}{1000}}{\frac{99}{100}} \right] \quad (2\frac{1}{2})$$

$$= \frac{2}{10} + \frac{63}{1000} \cdot \frac{100}{99}$$

$$= \left(\frac{1}{2} \right) \left[\frac{2}{10} + \frac{7}{110} \right] \quad (1)$$

$$= \left[\frac{29}{110} \right] \quad (1)$$

Use mathematical induction to prove that 3 is a factor of $n^3 + 6n^2 + 5n$ for all positive integers n .

SCORE: ____ / 10 PTS

BASIS STEP: 3 IS A FACTOR OF $1^3 + 6(1)^2 + 5(1) = 12$ ①

INDUCTIVE
STEP

① ASSUME 3 IS A FACTOR OF $k^3 + 6k^2 + 5k$

FOR SOME PARTICULAR BUT ARBITRARY INTEGER $k \geq 1$

MUST HAVE WORD

"INTEGER"

①

①

[PROVE 3 IS A FACTOR OF $(k+1)^3 + 6(k+1)^2 + 5(k+1)$]

$$\underline{(k+1)^3 + 6(k+1)^2 + 5(k+1)} \quad ①$$

$$= k^3 + 3k^2 + 3k + 1 \\ + 6k^2 + 12k + 6 \\ + 5k + 5$$

$$= (k^3 + 6k^2 + 5k) + (3k^2 + 15k + 12)$$

$$= \underline{(k^3 + 6k^2 + 5k) + 3(k^2 + 5k + 4)} \quad ②$$

SINCE 3 IS A FACTOR OF BOTH $(k^3 + 6k^2 + 5k)$ AND $3(k^2 + 5k + 4)$ ①

THEREFORE 3 IS A FACTOR OF $(k+1)^3 + 6(k+1)^2 + 5(k+1)$ ①

SO, 3 IS A FACTOR OF $n^3 + 6n^2 + 5n$ ①/2

FOR ALL INTEGERS $n \geq 1$ ①/2